Forecast Error Decomposition

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1. Introduction

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2. Forecast Error Decomposition
3. More Sophisticated Formulation

Forecast with error due to displacement

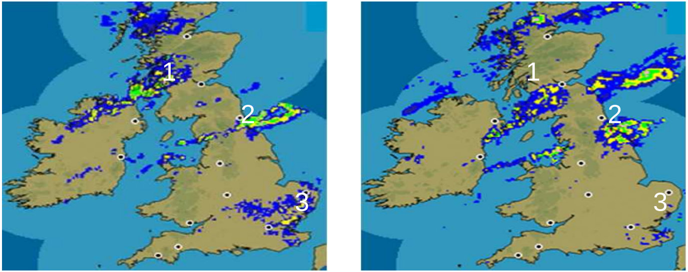

Figure: An illustration of the (a) Forecasted Data (b) Observed Data for rainfall on a given day

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Suppose q_1 represents the forecasted rain, and q_2 the rainfall that actually happened. We could calculate $||q_1 - q_2||$. But if the forecast got the qualitative features correct, but in the wrong place, $||q_1 - q_2||$ could be huge.

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In the case of displacement of a few intense storms covering a small area, the forecast would have been a good one.

Best fit of forecasted data

Figure: Forecasted Data

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Figure: Observed Data

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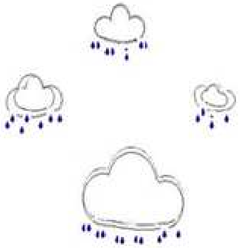

Figure: Best Fitting Data

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Total error for a weight $0 < \theta < 1$:

$$(1-\theta) \inf_{q \in \mathcal{R}(q_1)} \|q - q_2\|_2 + \theta \inf_{\hat{q} \in \mathcal{M}} W_2^2(q_1, \hat{q}).$$

Theoretical results

Theorem (Benamou, Brenier)

If you minimise "kinetic energy" over velocity fields which transport q_1 to $\hat{q}$, you get $W_2^2(q_1, \hat{q})$.

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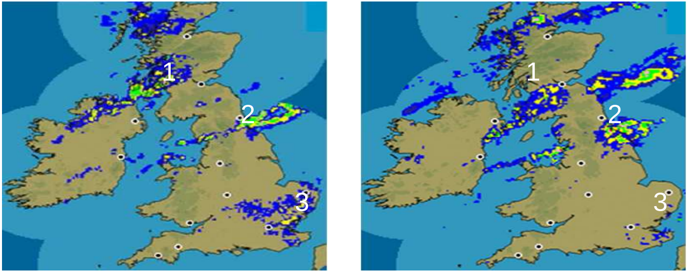
These theoretical results show how to construct numerical schemes to calculate qualitative features and displacement error.

Features error as displacement?



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(ii) More sophisticated formulation: minimise both quantities at the same time.

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- When comparing two quantities where displacement of features contributes significantly to the difference, forecast error decomposition gives a sensible answer regarding how close they are, and is descriptive of how they differ.

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- The well-posedness of a simple formulation has been demonstrated; work in progress on a more sophisticated scheme.

- Interested in analysis of PDEs which model the weather.

Some References

1. R.J. Douglas. Rearrangements of functions with applications to meteorology and ideal fluid flow in *Large-Scale Atmospheric-Ocean Dynamics I: analytical methods and numerical models*. Cambridge University Press, 288-341, 2002.
2. R.J. Douglas and G.D.E. Lanagan. Error Decomposition using rearrangements of functions. *Preprint*.
3. R.N. Hoffmann, Z. Liu, J.-F. Louis, and C. Grassotti. Distortion representation of forecast errors. *Mon. Wea. Rev.* **123**. 2758-2770, 1995.
4. C. Villani. *Topics in optimal transportation*. American Mathematical Society, 2003.